HOSSAM GHANEM

Asymptotes

V . A
Vertical Asymptotes

H . A
Horizontal Asymptotes

كيف تحصل على V.A للدالة $f(x)$

1 - أوجد أصفار مقام الدالة و ليكن a
(الدالة التي ليس لها أصفار مقام ليس له V.A)
2 - أوجد

الحالة الأولى
النهاية تساوي $(\pm\infty)$
يكون V.A موجود و معادلته هي $x = a$

الحالة الثانية
النهاية موجودة وتساوي عدد (c)
لا يوجد V.A
إذا كان الدالة لها أكثر من صفر للمقام
تكرر الخطوة رقم 2 مع كل صفر

أستخدم كل ما تعلمته عن
LIMITS INVOLVING $\pm\infty$

كيف تحصل على H.A للدالة $f(x)$

اوجد

الحالة الأولى
النهاية موجودة وتساوي عدد (a)
يكون H.A موجود و معادلته هي $y = a$

الحالة الثانية
النهاية تساوي $(\pm\infty)$
لا يوجد H.A

HOSSAM GHANEM

(9) 4.4 The Vertical And Horizontal Asymptotes (A)

Example 1	Let $f(x) = \dfrac{2x^3+5}{3x^3-24}$ Find the vertical and horizontal asymptotes for the graph f (if any)

Solution

$H.A$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2x^3+5}{3x^3-24} = \lim_{x\to\infty} \frac{2+\frac{5}{x^3}}{3-\frac{24}{x^3}} = \frac{2}{3}$$

$$\therefore y = \frac{2}{3} \quad H.A$$

$V.A$

$$3x^3 - 24 = 0$$

$$3(x^3-8) = 0$$

$$3(x-2)(x^2+4x+4) = 0 \quad \to\to\to \therefore \boxed{x=2}$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{2x^3+5}{2(x-2)(x^2+4x+4)} = -\infty$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{2x^3+5}{2(x-2)(x^2+4x+4)} = \infty$$

$$\therefore \ x = 2 \quad V.A$$

Example 2 43 June 28, 2008	Find the vertical and horizontal asymptotes f, if any , for $f(x) = \dfrac{(x^2-2x+1)\lvert x \rvert}{(x^2-1)x}$

Solution

$$f(x) = \frac{(x^2-2x+1)\lvert x \rvert}{(x^2-1)x} = \frac{(x-1)^2\,\lvert x\rvert}{(x-1)(x+1)\,x}$$

$H.A$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{(x-1)^2\,\lvert x\rvert}{(x-1)(x+1)\,x} = \lim_{x\to\infty} \frac{(x-1)^2\cdot x}{(x-1)(x+1)\,x} = \lim_{x\to\infty} \frac{(x-1)}{(x+1)} = \lim_{x\to\infty} \frac{1-\frac{1}{x}}{1+\frac{1}{x}} = 1$$

$$\therefore \ y = 1 \quad H.A$$

$$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{(x-1)^2\,(-x)}{(x-1)(x+1)\,x} = \lim_{x\to -\infty} \frac{-(x-1)}{(x+1)} = -1$$

$$\therefore y = -1 \quad H.A$$

$V.A$

$$x = 1 \qquad x = -1 \qquad x = 0$$

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} \frac{(x-1)^2\,(-x)}{(x-1)(x+1)\,x} = \lim_{x\to -1^-} \frac{-(x-1)}{(x+1)} = -\infty$$

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} \frac{(x-1)^2\,(-x)}{(x-1)(x+1)\,x} = \lim_{x\to -1^+} \frac{-(x-1)}{(x+1)} = \infty$$

$$\therefore \ x = -1 \quad V.A$$

Example 3 45 March 28, 2007	Let $f(x) = \dfrac{\lvert x-3\rvert(x+7)}{x^2-5x+6}$ Find the vertical and horizontal asymptotes for the graph f (if any)

Solution

$$f(x) = \frac{|x-3|(x+7)}{x^2-5x+6} = \frac{|x-3|(x+7)}{(x-3)(x-2)}$$

$H.A$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{|x-3|(x+7)}{(x-3)(x-2)} = \lim_{x\to\infty} \frac{(x-3)(x+7)}{(x-3)(x-2)} = \lim_{x\to\infty} \frac{(x+7)}{(x-2)} = \lim_{x\to\infty} \frac{\left(1+\frac{7}{x}\right)}{\left(1-\frac{2}{x}\right)} = 1$$

$$\therefore \ y = 1 \ \ H.A$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{-(x-3)(x+7)}{(x-3)(x-2)} = -1$$

$$\therefore y = -1 \ \ H.A$$

$V.A$

$$\lim_{x\to2^-} f(x) = \lim_{x\to2^-} \frac{-(x-3)(x+7)}{(x-3)(x-2)} = \infty$$

$$\lim_{x\to2^+} f(x) = \lim_{x\to2^+} \frac{-(x-3)(x+7)}{(x-3)(x-2)} = -\infty$$

$$\therefore \ x = 2 \ \ V.A$$

Example 4 47 November 10 .2007 A	Find the vertical and horizontal asymptotes for the graph f (if any) $f(x)=\frac{(x+1)\sqrt{x^2+2}}{x^2-x}$

Solution

$$f(x)=\frac{(x+1)\sqrt{x^2+2}}{x^2-x}=\frac{(x+1)\sqrt{x^2+2}}{x(x-1)}$$

$H.A$

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{(x+1)\sqrt{x^2+2}}{x^2-x}=\lim_{x\to\infty}\frac{(x+1)\cdot|x|\sqrt{1+\frac{2}{x^2}}}{x^2-x}=\lim_{x\to\infty}\frac{(x+1)\cdot(x)\sqrt{1+\frac{2}{x^2}}}{x^2-x}$$

$$=\lim_{x\to\infty}\frac{(x+1)\cdot(x)\sqrt{1+\frac{2}{x^2}}}{x^2-x}=\lim_{x\to\infty}\frac{(x^2+x)\sqrt{1+\frac{2}{x^2}}}{x^2-x}=\lim_{x\to\infty}\frac{\left(1+\frac{1}{x}\right)\sqrt{1+\frac{2}{x^2}}}{1-\frac{1}{x}}=1$$

$\therefore\ y=1\ \ H.A$

$\lim_{x\to-\infty} f(x)=-1$

$\therefore\ y=-1\ \ H.A$

$V.A$

$x(x-1)=0 \quad \to \quad x=0 \quad \& \quad x=1$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\frac{(x+1)\sqrt{x^2+2}}{x(x-1)}=\infty$$

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\frac{(x+1)\sqrt{x^2+2}}{x(x-1)}=-\infty$$

$\therefore\ x=0\ \ V.A$

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}\frac{(x+1)\sqrt{x^2+2}}{x(x-1)}=-\infty$$

$$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}\frac{(x+1)\sqrt{x^2+2}}{x(x-1)}=\infty$$

$\therefore\ x=1\ \ V.A$

Example 5	Let $f(x)=\dfrac{\sqrt[3]{x^9+3}}{2x^3-16}$ Find the vertical and horizontal asymptotes for the graph f (if any)

Solution

$$f(x)=\frac{\sqrt[3]{x^9+3}}{2x^3-16}=\frac{\sqrt[3]{x^9+3}}{2(x^3-8)}=\frac{\sqrt[3]{x^9+3}}{2(x-2)(x^2+2x+4)}$$

$H.A$

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{\sqrt[3]{x^9+3}}{2x^3-16}=\lim_{x\to\infty}\frac{\sqrt[3]{1+\frac{3}{x^9}}}{2-\frac{16}{x^3}}=\frac{1}{2}$$

$$\therefore\ y=\frac{1}{2}\qquad H.A$$

$V.A$

$$2(x-2)(x^2+2x+4)=0\qquad\to\qquad x=2$$

$$\lim_{x\to 2^+} f(x)=\lim_{x\to 2^+}\frac{\sqrt[3]{x^9+3}}{2(x-2)(x^2+2x+4)}=\infty$$

$$\lim_{x\to 2^-} f(x)=-\infty$$

$$\therefore\ x=2\qquad V.A$$

Example 6 48 March 25, 2008 A	Find the vertical and horizontal al asymptotes for the graph f (if any) where $f(x)=\dfrac{x^2+x\sqrt{x^2+1}}{(x+1)^2}$

Solution

$$f(x)=\frac{x^2+x\sqrt{x^2+1}}{(x+1)^2}=\frac{x^2+x\,|x|\sqrt{1+\frac{1}{x^2}}}{x^2+2x+1}$$

$H.A$

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{x^2+x^2\sqrt{1+\frac{1}{x^2}}}{x^2+2x+1}=\lim_{x\to\infty}\frac{1+\sqrt{1+\frac{1}{x^2}}}{1+\frac{2}{x}+\frac{1}{x^2}}=\frac{1+1}{1}=2$$

$$\therefore\ y=2\qquad H.A$$

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{x^2-x^2\sqrt{1+\frac{1}{x^2}}}{x^2+2x+1}=\lim_{x\to-\infty}\frac{1-\sqrt{1+\frac{1}{x^2}}}{1+\frac{2}{x}+\frac{1}{x^2}}=\frac{1-1}{1}=0$$

$$\therefore\ y=0\qquad H.A$$

$V.A$

$$x+1=0\qquad\to\qquad x=-1$$

$$\lim_{x\to-1^+} f(x)=\lim_{x\to-1^+}\frac{x^2-x^2\sqrt{1+\frac{1}{x^2}}}{(x+1)^2}=-\infty$$

$$\lim_{x\to-1^-} f(x)=\lim_{x\to-1^-}\frac{x^2-x^2\sqrt{1+\frac{1}{x^2}}}{(x+1)^2}=-\infty$$

$$\therefore\ x=-1\qquad V.A$$

Homework

Find the H.A and V.H Of f if

Find the vertical and horizontal asymptotes of f (if any)

1	$f(x)=\dfrac{\sqrt{x^2+4}}{x+4}$	41 March 30, 2005
2	$f(x)=\dfrac{2x+\sqrt{x^2+1}}{x+2}$	33 October 25,2001 A
3	$f(x)=\dfrac{2x-9}{x+3}$	
4	$f(x)=\dfrac{x^3-x}{x^2-x-2}$	37 July 12, 2003 A
5	$f(x)=\dfrac{\|x\|}{x^2-x}$	36 April 19,2003 A
6	$f(x)=\dfrac{\|x-1\|(1-2x)}{x^2+x-2}$	44 November 9, 2006
7	$f(x)=\dfrac{\|\sqrt{x}-2\|}{x-4}$	13 November 13, 1995
8	$f(x)=\dfrac{\|x+1\|}{x^2+x}$	39 July 3, 2004
9	$f(x)=\dfrac{x+3}{\sqrt{x}}$	
10	$f(x)=2-\dfrac{4}{x}+\dfrac{6}{x^2}$	
11	$f(x)=\dfrac{2\sqrt{x^2+7}}{x+5}$	
12	$f(x)=\dfrac{2x-9}{x+3}$	
13	$f(x)=\dfrac{x+3}{\sqrt{x}}$	

Homework

Find the H.A and V.H Of f if
Find the vertical and horizontal asymptotes of f (if any)

14	$f(x) = \dfrac{5x}{\sqrt{4x^2 + 3x}}$
15	$f(x) = \dfrac{\sqrt[3]{x^6 + 1}}{4x^2 - 1}$
16	$f(x) = \dfrac{5}{x^4 - 16}$
17	$f(x) = \dfrac{x + 2}{x^2 - 1}$
19	$f(x) = \dfrac{x^2 + 4}{x - 4}$
19	$f(x) = \dfrac{2\sqrt{x} + 5\lvert x\rvert}{x - 6}$
20	$f(x) = \dfrac{\lvert x - 5\rvert(x + 7)}{x^2 - 5x + 6}$
21	18 May 24 ,2000 $f(x) = \dfrac{\sqrt{x + 2} - 2}{x + 2}$
22	55 April 8, 2010 (3pts) Find the vertical and horizontal asymptotes, if any, for $y = \dfrac{\lvert x\rvert(2x^2 + 3)}{x^3 + 8}$
23	57 November 8, 2010 Find the vertical and horizontal asymptotes, if any, for the graph of (4 pts.) $f(x) = \dfrac{x\lvert x\rvert}{x^2 - x}$

Homework

24

12 November 2, 1995

Let

$$f(x) = \frac{4x^2 - 1}{(2x+1)\sqrt{4x^2+3}}$$

Find the vertical and horizontal asymptotes for the graph f (if any)

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47 November 10 2007 A

Find the vertical and horizontal asymptotes for the graph f (if any)

$$f(x) = \frac{(x+1)\sqrt{x^2+2}}{x^2 - x}$$

24 12 November 2, 1995	Let $f(x) = \dfrac{4x^2 - 1}{(2x+1)\sqrt{4x^2+3}}$ Find the vertical and horizontal asymptotes for the graph f (if any)

Solution

$$f(x) = \frac{4x^2 - 1}{(2x+1)\sqrt{4x^2+3}} = \frac{(2x+1)(2x-1)}{(2x+1)\,|x|\,\sqrt{4+\frac{3}{x^2}}}$$

$H.A$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{(2x-1)}{x\sqrt{4+\frac{3}{x^2}}} = \lim_{x\to\infty} \frac{\left(2-\frac{1}{x}\right)}{\sqrt{4+\frac{3}{x^2}}} = 1$$

$\therefore\ y = 1\ \ H.A$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{(2x-1)}{-x\sqrt{4+\frac{3}{x^2}}} = -1$$

$\therefore y = -1\ \ \ H.A$

$V.A$

$2x + 1 = 0 \quad \rightarrow \quad x = -\dfrac{1}{2}$

$$\lim_{x\to-\frac{1}{2}} f(x) = \lim_{x\to-\frac{1}{2}} \frac{(2x+1)(2x-1)}{(2x+1)\sqrt{4x^2+3}} = \lim_{x\to-\frac{1}{2}} \frac{(2x-1)}{\sqrt{4x^2+3}} = \frac{-2}{\sqrt{4\left(\frac{1}{4}\right)+3}} = -1$$

$\therefore\ NO\ \ \ V.A$

25 47 November 10 .2007 A	Find the vertical and horizontal asymptotes for the graph f (if any) $f(x)=\dfrac{(x+1)\sqrt{x^2+2}}{x^2-x}$

Solution

$$f(x)=\frac{(x+1)\sqrt{x^2+2}}{x^2-x}=\frac{(x+1)\,|x|\,\sqrt{1+\frac{2}{x^2}}}{x(x-1)}$$

$H.A$

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{(x+1)\,(x)\,\sqrt{1+\frac{2}{x^2}}}{x(x-1)}=\lim_{x\to\infty}\frac{(x+1)\,\sqrt{1+\frac{2}{x^2}}}{(x-1)}=\lim_{x\to\infty}\frac{\left(1+\frac{1}{x}\right)\,\sqrt{1+\frac{2}{x^2}}}{\left(1-\frac{1}{x}\right)}=1$$

$\therefore\ y=1\ \ H.A$

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{(x+1)\,(-x)\,\sqrt{1+\frac{2}{x^2}}}{x(x-1)}=-1$$

$\therefore y=-1\ \ H.A$

$V.A$

$x(x-1)=0 \quad \rightarrow \quad x=0 \quad \& \quad x=1$

$$\lim_{x\to1^+} f(x)=\lim_{x\to1^+}\frac{(x+1)\sqrt{x^2+2}}{x(x-1)}=\infty$$

$$\lim_{x\to1^-} f(x)=\lim_{x\to1^-}\frac{(x+1)\sqrt{x^2+2}}{x(x-1)}=-\infty$$

$\therefore\ x=1\ \ V.A$

$$\lim_{x\to0^+} f(x)=\lim_{x\to0^+}\frac{(x+1)\sqrt{x^2+2}}{x(x-1)}=-\infty$$

$$\lim_{x\to0^-} f(x)=\lim_{x\to0^-}\frac{(x+1)\sqrt{x^2+2}}{x(x-1)}=\infty$$

$\therefore\ x=0\ \ V.A$